

On the braided Fock spaces

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Abstract

Framework for constructing Fock spaces associated either with certain solutions of the quantum Yang-Baxter equation or with infinite dimensional Hecke algebra is presented. For the former case, the quantum deformed oscillator algebra associated with the solution of the quantum Yang-Baxter equation is found.

During last decade much attention have been paid to the quantum (q -deformed) groups and algebras (both will be called quantum groups below) as well as to their applications in such diverse branches of theoretical and mathematical physics as conformal field theory, integrable models, nuclear physics, statistical mechanics, knot theory and topological field theory (see, e.g., [1] and references therein). For the first time they appeared in fact as the main ingredient of quantum inverse scattering method [2, 3] and then were interpreted as Hopf algebras in [4, 5, 6] (see also [7] for the quantum matrix group interpretation). There exist various models describing systems of either one particle or several distinct particles, which possess quantum symmetry, i.e., are symmetrical with respect to the action of quantum group (see, e.g., [1], [8] - [10]). Unfortunately, the problem of description of a quantum symmetric (identical) multiparticle system is not solved completely yet. It is well known that quantum groups are deeply connected with the braid group, which substitutes usual symmetric group in this case [11, 12, 13] (see also [14]). Therefore, one can expect that to construct a multiparticle system possessing quantum symmetry, the braid group should be used instead of symmetric group. On the other hand, it is known that inequivalent quantizations of multiparticle systems on two-dimensional manifolds are labeled by the irreducible representations of the braid group, giving rise to braid statistics [15, 16, 17]. Being motivated by above-mentioned arguments, in this paper we construct the Fock spaces of particles obeying braid statistics.

Recent state of the problem is as follows. The Fock spaces of the particles obeying statistics associated with nontrivial representations of the symmetric group have been constructed in [18]. The same has been done in [19] for the multiparticle systems with the special emphasis on the quantum symmetrical properties of systems under consideration. In the number of papers Fock spaces are constructed, which originate from the creation and annihilation operators with nontrivial commutation relations being preserved under the action of some quantum groups [20] (see also [21]-[24]). The realization of the q -deformed Fock space in terms of q -wedges have been done in [25].

In the present paper, a framework for constructing the Fock space associated either with any solution of the quantum Yang-Baxter equation which obeys also the Hecke equation or with any representation of the infinite dimensional Hecke algebra $H_\infty(q^2)$ is developed. Here we deal with the Hecke algebra, which is a subalgebra of the group algebra of the braid group and, on the other hand, is q -deformation of the usual symmetric group. Other special realizations of the braid group e.g., Birman-Wenzel-Murakami algebra or truncated braid groups will be

considered elsewhere. Throughout this paper, q is generic complex number with the exception of root of unity.

Let us remind that the Hecke algebra $H_n(q^2)$ is defined in terms of the generators $\sigma_1, \sigma_2, \dots, \sigma_{n-1}$, which satisfy the following relations [13, 25]:

$$\begin{aligned} \sigma_i \sigma_{i+1} \sigma_i &= \sigma_{i+1} \sigma_i \sigma_{i+1}, & i = \overline{1, n-2}, \\ \sigma_i \sigma_j &= \sigma_j \sigma_i, & \text{if } |i-j| > 1, \\ (\sigma_i + 1)(\sigma_i - q^2) &= 0, & i = \overline{1, n-1}. \end{aligned} \quad (1)$$

First relation in (1) is (quantum) Yang-Baxter equation and third one is Hecke equation. The first and second relations in (1) are defining relations for the generators of the braid group. For $q = 1$ the relations (1) reduce to the generating relations of the symmetric group while the Hecke algebra reduces to the group algebra of the symmetric group.

There is a natural sequence of the Hecke algebras $H_k(q^2)$, $k = \overline{2, n}$:

$$H_2(q^2) \subset H_3(q^2) \subset \dots \subset H_n(q^2), \quad (2)$$

which are defined in terms of the generators $\sigma_{n+1-k}, \dots, \sigma_{n-1}$, respectively. The fundamental invariant of the Hecke algebra $H_k(q^2)$ is [13, 26]:

$$C_n^{(k)} = \sum_{j=n+1-k}^{n-1} L_j, \quad k = \overline{2, n}, \quad (3)$$

where L_j , $j = \overline{1, n-1}$, are the Murphy operators [26]:

$$L_{n-1} = \sigma_{n-1}, \quad (4)$$

$$L_j = \sum_{k=j}^{n-1} q^{-2(k-j)} \sigma_k \sigma_{k-1} \dots \sigma_{j+1} \sigma_j \sigma_{j+1} \dots \sigma_{k-1} \sigma_k, \quad j = \overline{1, n-2}. \quad (5)$$

Using (3)-(5) and the Hecke equation, one obtains [13, 26] that $C_n^{(k)}$ belongs to the center of $H_k(q^2)$:

$$[C_n^{(k)}, \sigma_j] = 0, \quad \text{for } j = \overline{n+1-k, n-1}, \quad (6)$$

and the relations hold

$$\begin{aligned} [L_{j_1}, L_{j_2}] &= 0, & j_1, j_2 &= \overline{1, n-1} \\ L_j &= q^{-2} \sigma_j L_{j+1} \sigma_j + \sigma_j, & j &= \overline{1, n-2} \\ L_j &= C_n^{(n+1-j)} - C_n^{(n-j)}, & j &= \overline{1, n-1}. \end{aligned} \quad (7)$$

Using relations (1)-(7) one obtains that the following propositions are valid.

Proposition 1 *The operators*

$$P_S^{(n)} = \frac{1}{[n]_{q^2}!} \prod_{j=1}^{n-1} (1 + L_j), \quad (8)$$

$$P_A^{(n)} = \frac{q^{n(n-1)}}{[n]_{q^2}!} \prod_{j=1}^{n-1} (1 - q^{-2} L_j), \quad (9)$$

have the following properties:

$$\sigma_i P_S^{(n)} = q^2 P_S^{(n)} , \quad i = \overline{1, n-1} , \quad (10)$$

$$\sigma_i P_A^{(n)} = -P_A^{(n)} , \quad i = \overline{1, n-1} , \quad (11)$$

$$\left(P_S^{(n)} \right)^2 = P_S^{(n)} , \quad (12)$$

$$\left(P_A^{(n)} \right)^2 = P_A^{(n)} , \quad (13)$$

and

$$P_S^{(n)} P_A^{(n)} = P_A^{(n)} P_S^{(n)} = 0 . \quad (14)$$

Here the following notations are used:

$$[n]_{q^2} = \frac{q^{2n} - 1}{q^2 - 1} , \quad [0]_{q^2}! = 1, \quad [n]_{q^2}! = \prod_{k=1}^n [k]_{q^2} , \quad n \geq 1. \quad (15)$$

Proposition 2 Let $H_n(q^2)$ be algebra with involution $*$. Then

$$\left(P_S^{(n)} \right)^* = P_S^{(n)} , \quad (16)$$

and

$$\left(P_A^{(n)} \right)^* = P_A^{(n)} , \quad (17)$$

if one of the following conditions satisfies

1) q^2 is real number, $\overline{q^2} = q^2$ and $(\sigma_i)^* = \sigma_i$, $i = \overline{1, n-1}$

or

2) $\overline{q^2} = q^{-2}$ and $(\sigma_i)^* = (\sigma_i)^{-1}$, $i = \overline{1, n-1}$, where

$$(\sigma_i)^{-1} = q^{-2} \sigma_i + (q^{-2} - 1) , \quad (18)$$

according to Hecke equation.

The above formulated propositions allow us to proceed to construction of the braided Fock spaces.

Let \mathcal{H} be either a separable Hilbert space or even a finite dimensional linear space with the orthonormal bases $\{e_k\}$, $k \in \mathcal{I}$, where \mathcal{I} is a set of nonnegative integers, $\mathcal{I} = \mathcal{N} \cup \{0\}$, for the first case and $\mathcal{I} = 1, 2, \dots, N$ (N is the dimension of \mathcal{H}) for the second one. Consider n -fold tensor power $\mathcal{H}^n = \mathcal{H}^{\otimes n}$ with the orthonormal bases $e_{k_1 k_2 \dots k_n} = e_{k_1} \otimes e_{k_2} \otimes \dots \otimes e_{k_n}$ and let $\mathcal{H}^0 = C^1$. Then the direct sum

$$\mathcal{F}(\mathcal{H}) = \bigoplus_{n=0}^{\infty} \mathcal{H}^n \quad (19)$$

is called the Fock space over \mathcal{H} .

Let us suppose that the action of the generators σ_i of the Hecke algebra $H_n(q^2)$ on the bases of \mathcal{H}^n , $n \geq 2$, is given by the formula:

$$\sigma_i e_{k_1 k_2 \dots k_n} = \sum_{l_i, l_{i+1} \in \mathcal{I}} \hat{\mathcal{R}}(k_i, k_{i+1}; l_i, l_{i+1}) e_{k_1 k_2 \dots k_{i-1} l_i l_{i+1} k_{i+2} \dots k_n} . \quad (20)$$

One can easily obtain from (1), (20) that $\hat{\mathcal{R}}$ -matrix satisfies the relations:

$$\begin{aligned} & \sum_{l_1, l_2, l_3 \in \mathcal{I}} \hat{\mathcal{R}}(m_1, m_2; l_1, l_2) \hat{\mathcal{R}}(l_2, m_3; l_3, n_3) \hat{\mathcal{R}}(l_1, l_3; n_1, n_2) = \\ & = \sum_{l_1, l_2, l_3 \in \mathcal{I}} \hat{\mathcal{R}}(m_2, m_3; l_2, l_3) \hat{\mathcal{R}}(m_1, l_2; n_1, l_1) \hat{\mathcal{R}}(l_1, l_3; n_2, n_3) , \end{aligned} \quad (21)$$

$$\sum_{l_1, l_2 \in \mathcal{I}} \hat{\mathcal{R}}(m_1, m_2; l_1, l_2) \hat{\mathcal{R}}(l_1, l_2; n_1, n_2) = (q^2 - 1) \hat{\mathcal{R}}(m_1, m_2; n_1, n_2) + q^2 , \quad (22)$$

(see, e.g. [27] for the examples of such $\hat{\mathcal{R}}$ -matrices). For action of σ_i , on the element $F^{(n)}$ of \mathcal{H}^n , $n \geq 2$

$$F^{(n)} = \sum_{k_1, k_2, \dots, k_n \in \mathcal{I}} F^{(n)}(k_1, k_2, \dots, k_n) e_{k_1 k_2 \dots k_n} , \quad (23)$$

one obtains from (20)

$$\sigma_i F^{(n)} = \sum_{k_1, k_2, \dots, k_n \in \mathcal{I}} e_{k_1 k_2 \dots k_n} \sum_{l_i, l_{i+1} \in \mathcal{I}} \hat{\mathcal{R}}(l_i, l_{i+1}; k_i, k_{i+1}) F^{(n)}(k_1, k_2, \dots, k_{i-1}, l_i, l_{i+1}, k_{i+2}, \dots, k_n) . \quad (24)$$

The space of sequences F :

$$F = \left(F^{(0)}, F^{(1)}(k_1), F^{(2)}(k_1, k_2), \dots, F^{(n)}(k_1, k_2, \dots, k_n), \dots \right) \quad (25)$$

with the scalar product

$$(F, \tilde{F}) = \sum_{n=0}^{\infty} (F^{(n)}, \tilde{F}^{(n)}) , \quad (26)$$

$$(F^{(n)}, \tilde{F}^{(n)}) = \sum_{k_1, k_2, \dots, k_n \in \mathcal{I}} \overline{F^{(n)}(k_1, k_2, \dots, k_n)} \tilde{F}^{(n)}(k_1, k_2, \dots, k_n) , \quad (27)$$

is isomorphic to $\mathcal{F}(\mathcal{H})$ and will be considered as $\mathcal{F}(\mathcal{H})$ henceforth. The action of σ_i on F follows from (24):

$$(\sigma_i F)^{(n)}(k_1, k_2, \dots, k_n) = \sum_{l_i, l_{i+1} \in \mathcal{I}} \hat{\mathcal{R}}(l_i, l_{i+1}; k_i, k_{i+1}) F^{(n)}(k_1, k_2, \dots, k_{i-1}, l_i, l_{i+1}, k_{i+2}, \dots, k_n) , \quad (28)$$

where $i < n$.

Let us turn now to the definition of the braided Fock spaces. Setting

$$P_S = \bigoplus_{n=0}^{\infty} P_S^{(n)} , \quad (29)$$

and

$$P_A = \bigoplus_{n=0}^{\infty} P_A^{(n)} , \quad (30)$$

where

$$P_S^{(0)} = P_S^{(1)} = P_A^{(0)} = P_A^{(1)} = I , \quad (31)$$

one obtains from Proposition 1 that P_S and P_A are projection operators (the operators $P_S^{(n)}$, $P_S^{(n)}$ for $n \geq 2$ are defined by formulas (8), (9) of Proposition 1). Furthermore, if one defines the action of involution $*$ by adjoint, i.e.

$$(\sigma_i)^* = (\sigma_i)^\dagger , \quad (32)$$

where $(\sigma_i)^\dagger$ is adjoint of σ_i , then it follows from Proposition 2 that P_S and P_A are orthogonal projectors, if either q^2 is real number and σ_i is selfadjoint operator or $\overline{q^2} = q^{-2}$ and σ_i is unitary operator. In what follows it will be assumed that the conditions of Proposition 2 hold.

The braided Fock spaces we are looking for are now defined by

$$\mathcal{F}_S(\mathcal{H}) = P_S \mathcal{F}(\mathcal{H}) , \quad (33)$$

$$\mathcal{F}_A(\mathcal{H}) = P_A \mathcal{F}(\mathcal{H}) . \quad (34)$$

According to Proposition 1, the $\mathcal{F}_S(\mathcal{H})$ and $\mathcal{F}_A(\mathcal{H})$ are the spaces of sequences

$$F_S = \left(F_S^{(0)}, F_S^{(1)}(k_1), F_S^{(2)}(k_1, k_2), \dots, F_S^{(n)}(k_1, k_2, \dots, k_n), \dots \right) , \quad (35)$$

$$F_A = \left(F_A^{(0)}, F_A^{(1)}(k_1), F_A^{(2)}(k_1, k_2), \dots, F_A^{(n)}(k_1, k_2, \dots, k_n), \dots \right) , \quad (36)$$

respectively, where $F_S^{(n)}(k_1, k_2, \dots, k_n)$ for $n \geq 2$ is an eigenelement of σ_i , $i = \overline{1, n-1}$ with eigenvalue q^2 and $F_A^{(n)}(k_1, k_2, \dots, k_n)$ for $n \geq 2$ is an eigenelement of σ_i , $i = \overline{1, n-1}$ with eigenvalue -1 . The above-mentioned means that $\mathcal{F}_S(\mathcal{H})$ and $\mathcal{F}_A(\mathcal{H})$ are Fock spaces of particles obeying braid statistics associated with certain solution $\hat{\mathcal{R}}$ of the Yang-Baxter equation.

To construct annihilation and creation operators on $\mathcal{F}_S(\mathcal{H})$ and $\mathcal{F}_A(\mathcal{H})$, let us first introduce on $\mathcal{F}(\mathcal{H})$ the operator $b(k)$, $k \in \mathcal{I}$ [28] by its action on $F \in \mathcal{F}(\mathcal{H})$:

$$\begin{aligned} (b(k) F)^{(0)} &= 0 , \\ (b(k) F)^{(n)}(k_1, k_2, \dots, k_n) &= F^{(n+1)}(k, k_1, k_2, \dots, k_n) , \quad n \geq 1 . \end{aligned} \quad (37)$$

The adjoint of $b(k)$, operator $b^\dagger(k)$ acts on $\mathcal{F}(\mathcal{H})$ by

$$(b^\dagger(k) F)^{(n)}(k_1, k_2, \dots, k_n) = \delta_{kk_1} F^{(n-1)}(k_2, \dots, k_n) . \quad (38)$$

Let us introduce following operators for the spaces $\mathcal{F}_S(\mathcal{H})$ and $\mathcal{F}_A(\mathcal{H})$ [28]:

$$b_S(k) = P_S b(k) P_S , \quad b_S^\dagger(k) = P_S b^\dagger(k) P_S , \quad (39)$$

$$b_A(k) = P_A b(k) P_A , \quad b_A^\dagger(k) = P_A b^\dagger(k) P_A . \quad (40)$$

The action of the operators $b_S(k)$ and $b_A(k)$ is given by

$$\begin{aligned} (b_S(k) F_S)^{(0)} &= 0 , \\ (b_S(k) F_S)^{(n)}(k_1, k_2, \dots, k_n) &= F_S^{(n+1)}(k, k_1, k_2, \dots, k_n) , \quad n \geq 1 , \end{aligned} \quad (41)$$

$$\begin{aligned} (b_A(k) F_A)^{(0)} &= 0 , \\ (b_A(k) F_A)^{(n)}(k_1, k_2, \dots, k_n) &= F_A^{(n+1)}(k, k_1, k_2, \dots, k_n) , \quad n \geq 1 . \end{aligned} \quad (42)$$

For the operators $b_S^\dagger(k)$ and $b_A^\dagger(k)$ formulas are more complicated:

$$\begin{aligned} (b_S^\dagger(k) F_S)^{(n)}(k_1, k_2, \dots, k_n) &= \left(\frac{1}{[n]_{q^2}} \right) \left\{ \delta_{kk_1} F_S^{(n-1)}(k_2, \dots, k_n) + \right. \\ &+ \sum_{l_2 \in \mathcal{I}} \hat{\mathcal{R}}(k, l_2; k_1, k_2) F_S^{(n-1)}(l_2, k_3, \dots, k_n) + \\ &+ \sum_{l_2, l_3, j_2 \in \mathcal{I}} \hat{\mathcal{R}}(l_2, l_3; k_2, k_3) \hat{\mathcal{R}}(k, j_2; k_1, l_2) F_S^{(n-1)}(j_2, l_3, k_4, \dots, k_n) + \\ &+ \sum_{m=3}^{n-1} \sum_{l_2, \dots, l_{m+1}, j_2, \dots, j_m \in \mathcal{I}} \hat{\mathcal{R}}(l_m, l_{m+1}; k_m, k_{m+1}) \left(\prod_{s=3}^m \hat{\mathcal{R}}(l_{s-1}, j_s; k_{s-1}, l_s) \right) \times \\ &\left. \times \hat{\mathcal{R}}(k, j_2; k_1, l_2) F_S^{(n-1)}(j_2, \dots, j_m, l_{m+1}, k_{m+2}, \dots, k_n) \right\} , \end{aligned} \quad (43)$$

$$\begin{aligned}
& \left(b_A^\dagger(k) F_A \right)^{(n)}(k_1, k_2, \dots, k_n) = \left(\frac{q^{2(n-1)}}{[n]_{q^2}} \right) \left\{ \delta_{kk_1} F_A^{(n-1)}(k_2, \dots, k_n) - \right. \\
& - q^{-2} \sum_{l_2 \in \mathcal{I}} \hat{\mathcal{R}}(k, l_2; k_1, k_2) F_A^{(n-1)}(l_2, k_3, \dots, k_n) + \\
& + q^{-4} \sum_{l_2, l_3, j_2 \in \mathcal{I}} \hat{\mathcal{R}}(l_2, l_3; k_2, k_3) \hat{\mathcal{R}}(k, j_2; k_1, l_2) F_A^{(n-1)}(j_2, l_3, k_4, \dots, k_n) + \\
& + \sum_{m=3}^{n-1} \left(-q^{-2} \right)^m \sum_{l_2, \dots, l_{m+1}, j_2, \dots, j_m \in \mathcal{I}} \hat{\mathcal{R}}(l_m, l_{m+1}; k_m, k_{m+1}) \left(\prod_{s=3}^m \hat{\mathcal{R}}(l_{s-1}, j_s; k_{s-1}, l_s) \right) \times \\
& \times \hat{\mathcal{R}}(k, j_2; k_1, l_2) F_A^{(n-1)}(j_2, \dots, j_m, l_{m+1}, k_{m+2}, \dots, k_n) \left. \right\}. \tag{44}
\end{aligned}$$

From equations (41)-(44) one gets:

$$\begin{aligned}
& \left(\left\{ b_S(k) b_S^\dagger(k') - \frac{[N]_{q^2}}{[N+1]_{q^2}} \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k', l; k, l') b_S^\dagger(l') b_S(l) \right\} F_S \right)^{(n)}(k_1, k_2, \dots, k_n) = \\
& = \frac{1}{[n+1]_{q^2}} \delta_{k k'} F_S^{(n)}(k_1, k_2, \dots, k_n), \tag{45}
\end{aligned}$$

$$\begin{aligned}
& \left(\left\{ b_A(k) b_A^\dagger(k') + \frac{[N]_{q^2}}{[N+1]_{q^2}} \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k', l; k, l') b_A^\dagger(l') b_A(l) \right\} F_A \right)^{(n)}(k_1, k_2, \dots, k_n) = \\
& = \frac{q^{2n}}{[n+1]_{q^2}} \delta_{k k'} F_A^{(n)}(k_1, k_2, \dots, k_n), \tag{46}
\end{aligned}$$

$$\begin{aligned}
& \left(\left\{ b_S^\dagger(k) b_S^\dagger(k') - q^{-2} \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k, k'; l, l') b_S^\dagger(l) b_S^\dagger(l') \right\} F_S \right)^{(n)}(k_1, k_2, \dots, k_n) = 0, \\
& \left(\left\{ b_S(k) b_S(k') - q^{-2} \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(l', l; k', k) b_S(l) b_S(l') \right\} F_S \right)^{(n)}(k_1, k_2, \dots, k_n) = 0, \tag{47}
\end{aligned}$$

$$\begin{aligned}
& \left(\left\{ b_A^\dagger(k) b_A^\dagger(k') + \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k, k'; l, l') b_A^\dagger(l) b_A^\dagger(l') \right\} F_A \right)^{(n)}(k_1, k_2, \dots, k_n) = 0, \\
& \left(\left\{ b_A(k) b_A(k') + \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(l', l; k', k) b_A(l) b_A(l') \right\} F_A \right)^{(n)}(k_1, k_2, \dots, k_n) = 0. \tag{48}
\end{aligned}$$

In (45), (46) N is the operator of a number of particles, which acts on $\mathcal{F}(\mathcal{H})$ by

$$(N F)^{(n)}(k_1, k_2, \dots, k_n) = n F^{(n)}(k_1, k_2, \dots, k_n). \tag{49}$$

It is interesting to note that equations (45)-(48) are obtained from relations (41)-(44) without any assumptions about selfadjointness of the projectors P_S and P_A .

Now one can define the creation and annihilation operators for the spaces $\mathcal{F}_S(\mathcal{H})$ and $\mathcal{F}_A(\mathcal{H})$:

$$\begin{aligned} a_S(k) &= \sqrt{[N+1]_{q^2}} b_S(k) , \\ a_S^\dagger(k) &= b_S^\dagger(k) \sqrt{[N+1]_{q^2}} , \end{aligned} \quad (50)$$

$$\begin{aligned} a_A(k) &= q^{-N} \sqrt{[N+1]_{q^2}} b_A(k) , \\ a_A^\dagger(k) &= b_A^\dagger(k) q^{-N} \sqrt{[N+1]_{q^2}} . \end{aligned} \quad (51)$$

The operators $a_S(k)$ and $a_S^\dagger(k)$ are adjoint if the second condition of the Proposition 2 satisfies. The same is valid for the operators $a_A(k)$ and $a_A^\dagger(k)$. Following commutation relations do not depend on the above-mentioned condition of adjointness:

$$a_S(k) a_S^\dagger(k') - \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k', l; k, l') a_S^\dagger(l') a_S(l) = \delta_{k k'} , \quad (52)$$

$$a_S^\dagger(k) a_S^\dagger(k') - q^{-2} \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k, k'; l, l') a_S^\dagger(l) a_S^\dagger(l') = 0 , \quad (53)$$

$$a_S(k) a_S(k') - q^{-2} \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(l', l; k', k) a_S(l) a_S(l') = 0 , \quad (54)$$

$$a_A(k) a_A^\dagger(k') + q^{-2} \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k', l; k, l') a_A^\dagger(l') a_A(l) = \delta_{k k'} , \quad (55)$$

$$a_A^\dagger(k) a_A^\dagger(k') + \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(k, k'; l, l') a_A^\dagger(l) a_A^\dagger(l') = 0 , \quad (56)$$

$$a_A(k) a_A(k') + \sum_{l, l' \in \mathcal{I}} \hat{\mathcal{R}}(l', l; k', k) a_A(l) a_A(l') = 0 . \quad (57)$$

Operators $a_S(k)$ ($a_A(k)$) and $a_S^\dagger(k)$ ($a_A^\dagger(k)$) with commutation relations (52)- (54) ((55)-(57)) are known as quantum deformed oscillator algebra.

Relations (53), (56) are preserved under the linear transformations

$$a^\dagger(k) \rightarrow \sum_{l \in \mathcal{I}} T(k, l) a^\dagger(l), \quad k \in \mathcal{I} , \quad (58)$$

where $a^\dagger(k)$ denote either $a_S^\dagger(k)$ or $a_A^\dagger(k)$ and $T(i, j)$, $i, j \in \mathcal{I}$ obey relations:

$$\sum_{p, s \in \mathcal{I}} T(i, p) T(j, s) \hat{\mathcal{R}}(p, s; k, l) = \sum_{p, s \in \mathcal{I}} \hat{\mathcal{R}}(i, j; p, s) T(p, k) T(s, l) , \quad (59)$$

$$T(i, j) a^\dagger(k) = a^\dagger(k) T(i, j) , \quad i, j, k \in \mathcal{I} . \quad (60)$$

Similarly, relations (54), (57) are preserved under the linear transformations

$$a(k) \rightarrow \sum_{l \in \mathcal{I}} T^*(k, l) a(l), \quad k \in \mathcal{I} , \quad (61)$$

where $a(k)$ denote either $a_S(k)$ or $a_A(k)$ and $T^*(i, j)$, $i, j \in \mathcal{I}$ obey relations:

$$\sum_{p, s \in \mathcal{I}} T^*(i, p) T^*(j, s) \hat{\mathcal{R}}(l, k; s, p) = \sum_{p, s \in \mathcal{I}} \hat{\mathcal{R}}(s, p; j, i) T^*(p, k) T^*(s, l) , \quad (62)$$

$$T^*(i, j)a(k) = a(k)T^*(i, j) , \quad i, j, k \in \mathcal{I} . \quad (63)$$

One has not mixed designation $*$ in (62), (63) with the involution of the Hecke algebra introduced in Proposition 2.

Relations (59) define quantum algebra associated with $\hat{\mathcal{R}}$ -matrix $\hat{\mathcal{R}}(i, j; k, l)$ while relations (62) define quantum algebra associated with transposed $\hat{\mathcal{R}}$ -matrix $\hat{\mathcal{R}}^*(i, j; k, l) = \hat{\mathcal{R}}(l, k; j, i)$.

Let us close this paper with some concluding remarks. Starting from the representation (20), (28) of the Hecke algebra, we built the Fock space of particles, which obey the braid statistics associated with a certain solution $\hat{\mathcal{R}}$ of the Yang-Baxter equation. Braid statistics nature of the Fock spaces under consideration is obvious from the commutation relations (52)-(57). It follows from Propositions 1 and 2 that the framework developed in the paper allows us to build Fock spaces associated with infinite dimensional Hecke algebra $H_\infty(q^2)$ acting on Fock space over \mathcal{H} , $\mathcal{F}(\mathcal{H})$ instead of one associated with $\hat{\mathcal{R}}$.

It is interesting to note that there does not exist nontrivial deformation of the algebra of oscillators within the category of algebras [29] at least for a finite dimensional case. It means that any deformation of the oscillator algebra can be considered as a deformation map from a undeformed algebra. There exist number of examples for such deformation maps [30]. From the other hand, being separable Hilbert spaces, the Fock spaces constructed in this paper and usual (anti)symmetric Fock space are isomorphic. It seems to be interesting to investigate connections between such isomorphisms and the above-mentioned deformation map. Another interesting problem arises due to a so called quon algebra [31], for which the associated Fock space can be constructed also. To obtain such a Fock space as one associated with the braid group is a task of future investigations.

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